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On the Orbit of 70 *Ophiuchi.* By Eyre B. Powell, Esq.

"Having been for some time engaged in investigating the orbit of the companion to 70 *Ophiuchi*, I have, at length, arrived at an ellipse which represents with considerable accuracy the angular motion of that interesting binary star; and I now do myself the pleasure of communicating my results to the Society.

"The process by which I obtained a first approximate orbit was the graphical one invented by Sir J. Herschel: three corrections then gave me the following elements:—

$$\tau = 1806{\cdot}82$$
$$\pi = 291^\circ\,40'$$
$$\gamma = 46\ 2$$
$$\Omega = 294$$
$$e = {\cdot}566$$
$$n = -3^\circ{\cdot}676, \text{ and therefore } P = 97{\cdot}93 \text{ years.}$$

"The errors of position afforded by these elements for 1779, 1804, 1825, 1830, 1833, 1842, and 1852, are all moderate, scarcely ever exceeding a degree; but the discrepancy between the observed and computed positions for 1802 reaches the enormous amount of eight degrees. It is worthy of notice that, while the angle for 1802 is so erroneous, the position for 1804 differs by only 8′ from the result of observation.

"The subjoined table contains the variations of the angles of

position, at eight different epochs, corresponding to changes in the preceding elements:—

Table of Variations of Position.

(Variations in minutes of space.)

Epoch.	τ	π	γ	Ω	e	n
1779·77	$+\frac{33a}{4}$	$+\frac{71b}{60}$	$+21k$	$-\frac{67d}{5}$	$+\frac{61f}{2}$	$+\frac{25c}{6}$
1802·34	$+\frac{281a}{4}$	$+\frac{49b}{30}$	$-\frac{73k}{2}$	$-\frac{156d}{5}$	$+125f$	$+\frac{25c}{2}$
1804·42	$+\frac{257a}{4}$	$+\frac{72b}{60}$	$-\frac{7k}{2}$	$-\frac{41d}{5}$	$+69f$	$+\frac{25c}{6}$
1825·56	$+14a$	$+\frac{81\cdot5b}{60}$	$-33k$	$-\frac{79d}{5}$	$-53f$	$-25c$
1830·5	$+9\cdot5a$	$+\frac{71b}{60}$	$-27k$	$-7d$	$-37f$	$-\frac{25c}{2}$
1833·5	$+\frac{33a}{4}$	$+\frac{68b}{60}$	$-\frac{47k}{2}$	$-5d$	$-30f$	$-\frac{25c}{2}$
1842·55	$+\frac{23a}{4}$	$+\frac{62b}{60}$	$-\frac{27k}{2}$	$-\frac{3d}{5}$	$-15f$	$-\frac{25c}{3}$
1852·75	$+5a$	$+b$	$-3k$	$+\frac{2d}{5}$	$-\frac{13f}{4}$	$-\frac{25c}{2}$

a being tenths of a year; b, minutes of space; k, degrees; d, degrees; f, hundredths of unity; and c, minutes of space.

"The inspection of the variations thus drawn together affords hints with regard to the management of the equations of correction for the different elements. Such equations were formed and combined in a variety of ways, so as to make the values of the corrections depend chiefly on those epochal equations in which the corresponding coefficients were largest. The method of least squares is commonly used to attain the greatest accuracy; but it seemed to me that the foregoing plan would be sufficient to answer the purpose in hand. Thus, then, new and more correct values of the elements were found; but, as a comparison of the angles of position afforded by them, with those of observation, was not so satisfactory as I wished, I again altered some of the elements, and finally adopted the following:—

$$\tau = 1806\cdot92$$
$$\pi = 291^\circ\,40'$$
$$\gamma = 49\ 56$$
$$\Omega = 296\ 30 \text{ and } \lambda = 7^\circ\,29'$$
$$e = \cdot546$$
$$n = -3^\circ\cdot668, \text{ and therefore } P = 98\cdot146 \text{ years.}$$

"The length of the period is, I believe, considerably greater than any hitherto obtained; but an examination of the angular velocities of the star bears out my result.

From 1833	to 1853	ϑ Position	= about	19
1842	— 1852	——	= —	$8\frac{1}{3}$
1846·2	— 1852·7	——	= —	6

"Now, to complete a whole revolution, so as to reach a position angle of 90°, about $23°\frac{1}{2}$ have to be described; and this will require at least 23 years. Hence, as 74 years have already elapsed, the periodic time cannot differ much from 98 years, and is, probably, somewhat greater.

"I annex a comparison of the observed angles of position with those given by the preceding elements.

Table of Comparisons of Observed and Computed Positions.

Date.	Position Observed. ° ′	Position Computed. ° ′	$P_o - P_o$. ° ′	Observer.
1779·77	90 0	89 4	— 56	$\mathrm{H}_{\odot}$
1802·34	334 38	336 10	+1 32	$\mathrm{H}_{\odot}$
1804·42	318 40	315 27	−3 13	$\mathrm{H}_{\odot}$
1821·72	157 39	157 19	— 20	Σ
1822·6	153 54	154 25	+ 31	Σ
1825·56	148 12	146 36	−1 36	Σ & S
1828·67	140 18	140 25	+ 7	Σ & H
1830·5	137 28	137 25	— 3	B, H, & D
1832·561	134 21	134 26	+ 5	H & D
1835·56	130 36	130 37	+ 1	Sm
1838·51	126 30	127 19	+ 49	Sm
1842·55	122 24	123 17	+ 53	Sm
1846·017	120 31	120 7	— 24	J
1850·571	116 6	116 12	+ 6	J & F
1852·75	114 3	114 22	+ 19	J
1854·081	113 39	113 16	— 23	J

$\mathrm{H}_{\odot}$, Sir W. Herschel; Σ, Professor Struve; S, Sir J. South; H, Sir J. Herschel; B, Professor Bessel; D, the Rev. Mr. Dawes; Sm, Admiral Smyth; J, Captain Jacob; F, Mr. Fletcher.

"The epochs pitched upon for the comparison were generally those for which there existed the most trustworthy observations; and in making the selection, I was guided, not only by the circumstances recorded of the measures, but by the angular velocities the latter afforded. Also, in combining the values assigned by different observers, I took the liberty of allotting weights according to the coherence and number of the sets of measures. The position for 1850 is a simple arithmetical mean between the angles of Captain Jacob and Mr. Fletcher, which differ *inter se* by nearly two degrees. I am inclined to think that Captain Jacob's angle for 1854 is rather too high; first, as the angular velocity obtained by combining it with his measure for 1852 is only three-tenths of a degree; and secondly, as I made the position for

1853·586, by 59 fairly accordant observations, equal to 113° 35′, which is but 5′ less than the result of calculation.

"Using the elements and the measured distances for 1781, 1825, 1828, 1830, 1832, 1835, 1838, 1842, and 1850, I computed the corresponding values of a, the semi-axis major; the mean came out 4″·48.

"With this value of a the following table was formed, showing the comparison of the calculated and observed distances for thirteen epochs. The inclination of the true distance to the apparent one is included in the table, as it may be satisfactory to see its effect in fore-shortening the radius vector. The formulæ used were

$$\sin I = \sin\gamma \sin(v \pm \lambda), \quad \varrho = a(1 - e\cos u), \text{ and } r = \varrho \cos I.$$

Table of Comparison of Observed and Computed Distances.

Date.	ϱ, or Radius Vector.	I, or Inclination.	r_c, or Comp. Dist.	r_o, or Observ. Dist.	$r_c - r_o$.	Observer.
		° ′				
1781·74	5·687	31 9	4·867	4·49	+·377	H$_S$
1804·42	2·177	21 6	2·031	2·56	−·529	H$_S$
1825·56	4·873	30 48	4·186	4·00	+·186	Σ
1828·67	5·288	25 44	4·764	4·79	−·026	Σ
1830·5	5·509	23 0	5·071	5·478	−·407	B, Sm, & D
1832·561	5·736	20 6	5·386	5·456	−·070	H & D
1835·56	6·028	16 10	5·79	5·97	−·180	Sm
1838·51	6·273	12 35	6·123	6·351	−·228	B, Sm, & D
1842·55	6·543	7 59	6·48	6·682	−·202	Sm & D
1846·017	6·716	4 18	6·697	6·83	−·133	J
1850·571	6·864	0 30	6·864	6·86	+·004	J
1852·75	6·904	2 31	6·897	6·73	+·167	J
1854·081	6·918	3 50	6·903	6·365	+·538	J

"On glancing at the numbers in the sixth column, the arrangement of the signs of $r_c - r_o$ suggests the existence of error in the elements. It has been asserted that the distance is now decreasing; if this be correct, the node, or the peri-astre, or both, should be somewhat advanced. As, however, the measures of distance are undoubtedly less worthy of confidence than those of position, it appears unnecessary at present to modify the elements with the sole view of reconciling the distances. No weight can be attached to the distance of 1804, as it was obtained by mere estimation in diameters. In 1825 Professor Struve found the projected radius vector 4″·0, Sir J. South 4″·76; the true value in all probability lies between these determinations, and, perhaps, nearer the former than the latter. This opinion is grounded upon two facts; first, that on Professor Struve's testing his distances, he found them on an average about two-tenths of a second too small; and secondly, that the distance in 1828 was only 4″·79, while

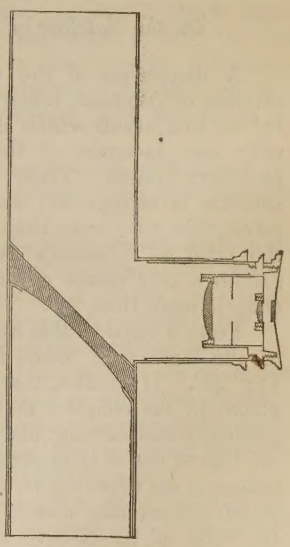
from the nature of the orbit the value of r was increasing pretty rapidly. For 1850 Captain Jacob and Mr. Fletcher differ by $0''{\cdot}366$, and the distance by computation agrees nearly with that obtained by the former. The most offensive discrepancy in the table is that for the current year; but even here it seems probable the orbit is not so far in fault as might at first be fancied. The angular velocity is apparently as small, or even smaller, than it was prior to 1850, and the radius vector lies near both the axis major and the line of nodes; hence there can be but a slight change in the distance.

"Upon the whole, the preceding elements appear to represent the motion of 70 *Ophiuchi* tolerably well; and it is to be feared that, till the companion approaches its peri-astre, but little can be done to fix definitively the most important points connected with the orbit. Probably, however, three or four years' farther observations will put it in our power to improve the position of the ellipse with reference to the line of nodes.

"*Madras, August 9th*, 1854."

Description of an Eye-piece for Observing the Sun.

By R. Hodgson, Esq.



"The novelty in the arrangement of this eye-piece is, that it permits the full aperture of the object-glass to be used while observing the whole of the sun's disk; an object of importance in solar eclipses and transits of planets across the sun; and it has also the further advantage of allowing the application of all the usual eye-pieces and micrometers.

"It consists in applying to the draw-tube of the telescope a diagonal glass reflector, about two inches in advance of the focal point; the first or reflecting surface being perfectly flat, and the second ground to a concave figure, and left unpolished: a dark glass is, of course, still necessary to protect the eye from the glare of the sun; but a light neutral tint is sufficient.

"I have applied it to my 6-inch equatoreal, and also tried it with equal success upon a telescope of 8 inches' aperture.

"*November* 1854."

Note respecting the recent Pendulum Experiments in the Harton Colliery. By G. B. Airy, Esq., Astronomer Royal.

Since the last meeting of the Society, the computations of the pendulum experiments in the Harton colliery have been completed, and the result appears very satisfactory. The pendulums do not appear to have received the slightest injury or change during the operations of interchange of position. The final result for the pendulums is,—

Acceleration at the bottom of the mine is 2·25 vibrations per day; or,

Increase of gravity at the bottom of the mine is $\frac{1}{19190}$ part of the whole.

Surveys will now be made for obtaining those data which are necessary in order to deduce, from this first result, the final result as to the earth's density. Until these are finished, no approximate result can be given with confidence: it appears likely, however, that the resulting density of the earth will be greater than those hitherto obtained.

December 8, 1854.

On the Satellite of Neptune. By J. R. Hind, Esq.

A discussion of the valuable series of observations of the satellite of *Neptune*, taken at Valetta in 1852, by Mr. Lassell, has led me to a result which may not be without interest, as hitherto only one instance of the kind was known to occur in the planetary system. There can be no doubt that the motion of the satellite is *retrograde;* and it offers a far more decided case of retrograde movement than the Uranian system, the inclination of the orbit of *Neptune's* attendant being only 29°, whereas the satellites of *Uranus* revolve in a plane which is inclined to the ecliptic more than 79°.

From a considerable number of observations I found the place of the ascending node, in the retrograde orbit, in longitude 175° 40′. It follows, therefore, from the present position of the planet in the ecliptic, that the apparent orbit of the satellite is gradually contracting, and will become a right line in 1859, when the line of nodes is directed towards the earth. Mr. Lassell has informed me that this is confirmed by his recent observations, in which the satellite was found to pass nearer to the planet than at the time of its discovery—a circumstance irreconcilable with *direct* movement, since, in that case, the place of the node would be behind the present position of *Neptune* on the ecliptic, and the apparent orbit of the satellite would necessarily be opening.

Another inference may be drawn from the observations of

1852, namely, the existence of a very considerable ellipticity in the orbit, though, unfortunately, the determination of its precise amount, as well as a fair approximation to the position of the line of apsides, are scarcely practicable from present data; and since the apparent ellipse is rapidly approaching to a right line, we must wait 12 or 14 years before the measures necessary for completing the investigation can be obtained.'

It will therefore be understood, that the position of the peri-neptunium and the angle of eccentricity in the following elements are very uncertain, though the inclination and node, and also the period of revolution, are, I believe, pretty exact.

Epoch 1852, November 0·0, Greenwich M.T.

Mean Anomaly	243° 32′
Peri-neptunium	177 30
Ascending Node	175 40
Inclination	151 0
Angle of Eccentricity	6 5
Period	5ᵈ·8769

If I am right in placing the lines of nodes and apsides nearly in coincidence, it will follow that, at the present date, there will be no very sensible difference between the extreme distances of the satellite eastward and westward of the primary, and this deduction appears to be borne out by a comparison of the measured distances at those points. It will also necessarily result that the satellite must pass nearer to the planet, when it is in the north-preceding quadrant, than it does in the opposite direction—a condition agreeing well with observation. At this epoch the satellite is nearest to the earth when it is to the north of its primary.

I subjoin an extract from a communication with which I was favoured by Mr. Lassell, affording strong evidence in support of the conclusions deduced above:—

"I certainly incline to the opinion that the apparent orbit of *Neptune's* satellite has diminished its conjugate axis since the earlier observations, though all but those of 1852 are made so near the greatest elongation that they could scarcely give even a rude approximation to the true form of the apparent orbit. * * I don't know whether it has occurred to you that the observations at Malta seem to indicate pretty clearly an apparent eccentricity of the orbit,—the satellite passes the planet at its nearest approach on the north-preceding side, something like a second *nearer* than at its nearest approach on the south-following side."

The semi-major axis of the satellite's orbit, as seen at the mean distance of *Neptune*, subtends an angle of 16″·98, according to a preliminary discussion of Mr. Lassell's Valetta measures, whence, with the periodic time I have given above, the mass of the planet is found to be $\frac{1}{17135}$, and the mean distance of the

satellite from the centre of *Neptune*, 235,800 miles. I believe it will appear, from a comparison of this gentleman's measures generally with those of other observers, that they have a tendency to be in excess rather than in defect of the mean, and consequently the value of the mass, deduced from them, will possibly admit of some diminution: probably $\frac{1}{17500}$ will prove very near the truth.

December 7, 1854.

Note on the Satellites and Mass of Uranus. By J. R. Hind, Esq.

In the *Monthly Notice* of this Society for March 11th, 1853, appears an extensive set of measures of the two brighter satellites of *Uranus*, *Titania* and *Oberon*, taken by Mr. Lassell during his residence at Malta, under circumstances far superior to those which affect the accuracy of such observations in the climate of this country. I have lately worked up those measures with the view of determining the positions of the orbits of the satellites and the mass of their primary. My results are as follow:—

I. *Oberon.*

Radius of Orbit at the Mean Distance of Uranus ..... 45″·20
[or 385,000 miles]

	° ′
Ascending Node	165 28
Inclination	100 34

II. *Titania.*

Radius of Orbit at the Mean Distance of Uranus ..... 33″·88
[or 288,600 miles]

	° ′
Ascending Node	165 25
Inclination	100 34

It will be remarked that these observations give the position of the node and inclination almost precisely the same as they were found by Sir William Herschel more than half a century ago—his numbers being $\Omega = 165° 30'$, $i = 101° 2'$.

I find from the distances of *Oberon*, mass of *Uranus* $= \frac{1}{20642}$, and from those of *Titania* $\frac{1}{20505}$, adopting Mr. Adams' periods of revolution: the mean $\frac{1}{20570}$ agrees very nearly with the value derived from the measures of Sir W. Herschel.

Mr. Lassell's observations indicate with tolerable certainty that the mass of *Neptune* is greater than that of *Uranus* in the proportion of 7 to 6.

1854, *Dec.* 7.

United States Expedition to Chili.

(*Extract of a Letter from Lieut. G. M. Gilliss, U.S.N. to Admiral W. H. Smyth.*)

"Preparatory to the publication of our volume of *Differential Observations on Mars and Venus in Chili*, we are discussing our longitude, and need for this purpose all the corresponding moon-culminations in Europe.

"Congress acted very liberally in the order to publish our results, directing that the whole work should be well printed on good paper, of quarto size, and neatly bound. In all there will be five volumes. Vol. i., containing a full account of Chili (except its political history), and Lieut. MacRae's narrative of the magnetical expedition across the Andes and Pampas of Buenos Ayres, will fill more than 700 pages. It has 51 maps and plates of landscapes, natural history, &c. These are in the engraver's hands, and the volume will probably be ready in July.

"Vol. ii., the differential observations on *Mars* and *Venus* wherever made and furnished us. We shall probably fill 500 pages, and the volume will be put to press by May.

"Vols. iii. and iv., meridian observations and zones, each 500 to 600 pages. The instrumental and clock errors have been computed for a large portion of the time; but it is not probable the volumes will be printed in less than $2\frac{1}{2}$ years. The reduction of more than 20,000 *new* stars and their arrangement in the order of right ascension from the zones is a work of much time.

Vol. v., magnetical and meteorological observations, will fill also 500 or 600 pages. This will appear shortly after vol. ii.

"You will perceive that we three in Chili accomplished no small amount of labour; I hope the results may prove acceptable to men of science.

"*Washington, 8th Nov.*, 1854."

On the Resistance of the Ethereal Medium and the Attraction of the Small Planets. By A. J. Angström, Esq., Junior Professor of Astronomy in the University of Uppsala.*

The planetary secular perturbations have more especially engaged the attention of astronomers, from the circumstance that the solution of questions, respecting the stability of our solar system,

* This paper is a translation from the Swedish, by A. D. Wackerbarth, Esq. who also communicated it to the Society.

finally depends on the amount of these variations, which, however inconsiderable they may appear, may, in the lapse of ages, accumulate to such a degree, as entirely to change the character of the elements.

Both theory and experience, however, agree that perturbations of such a nature cannot take place within our planetary system. The exception to this rule, which, in the beginning of the present century, was supposed to have been found in the moon's motion, whereof the mean motion had, from the earliest period to which astronomical observations extend, continually increased, disappeared when Laplace succeeded in explaining that phenomenon by a slow change in the eccentricity of the earth's orbit. Hence it appeared that this variation was, like its cause, periodical, and did not even affect directly the mean velocity, but what is called the epoch.

The case is, however, otherwise with comets. Here the largeness of the inclinations and eccentricities by no means excluded perturbations, even of a periodic nature, of such extent as entirely to change the elements of the orbit. A striking example of this is afforded by Lexell's comet, and, probably, also the two comets named after Brorsen and D'Arrest. And such must generally be the case with any comet that comes into the neighbourhood of *Jupiter*. But if the comets can be disturbed in their orbits by the planetary attractions, on the other hand, the masses of the comets are so trifling, that they cannot exercise any sensible influence on the motions of the planets.

Nevertheless, one of the comets, namely that of Encke, has given rise to a supposition which, should it be confirmed, must eventually make itself evident in the motions of the planets, and form a real exception to the received law of the unchangeableness of the mean distances, as subjected only to periodic variations.

It has, in fact, appeared that this comet — independently of the considerable changes which the period undergoes from the planetary perturbations—has, at each revolution, increased its mean velocity in its orbit, so as each time to arrive at perihelion $2\frac{1}{2}$ hours earlier than on the preceding occasion.

This circumstance is easiest explained in the following manner, according to the evidence produced by the learned calculator of that comet, Professor Encke:—there enters, or may be supposed to enter, into the expression for the mean longitude, a term varying as the square of the time; but then comes the difficulty of accounting for the presence of that term by the general law of gravitation. This has not hitherto been done; and Encke has accordingly had recourse to another means of explanation, namely, the resistance of an ethereal medium.

It is known that the effect which a resisting medium, of the same nature as the atmosphere, would have on a heavenly body's motion, would be to reduce the periodic time and eccentricity of the orbit, whereby there must arise an augmentation in the mean velocity, continually increasing with the time. This was just

what was wanted; and, on the strength of this experience, Encke assumed, in speciality for his ether, that its density decreases as the square of the distance from the sun; and, furthermore, that the resistance is proportional to the density of the ether and the square of the comet's velocity.

This supposition is, however, subject to weighty objections. First and principally, if the density of the medium increases up to the surface of the sun, it must hence follow that the ether itself participates in the solar system's progressive motion, as well as in the sun's rotation; for we have no right to ascribe to the ether the properties of air in one instance, and deny the consequences thereof in another. The sun's rotatory motion is then immediately communicated to the circumjacent condensed ether, and is thence communicated from stratum to stratum, with either the same or a diminished angular velocity. If we assume that the angular velocity diminishes in the same ratio as that of the planets, it follows that the resistance to the planets and to the comets of direct motion is nearly evanescent, but is so much the more increased in the case of comets with retrograde motion. But that this is not the case is sufficiently evidenced by Halley's comet.

The apparent diminution in the volume of the nucleus of Halley's comet, when in the neighbourhood of the sun, has been taken as evidence of the ether's increasing density. But as ether possesses the property of passing freely through all bodies, an increase in its density would not compress, but, on the contrary, if it had any effect at all, would expand bodies. The Newtonian theory for the origin of comets' tails, namely, that they are turned from the sun for the same reason as vapours rise in our own atmosphere, Bessel* has shown to be insufficient to account for the various changes in their form.

Since, then, on the one hand, on the hypothesis of the ether's condensation, its resistance, in the case of Encke's comet, must vanish, and, on the other, that hypothesis is insufficient to explain the phenomena as desired, it would seem that it ought to be rejected.

On the density of ether within diaphanous bodies, opinions, as we know, differ; since some have considered that density as constant, while others again, like Fresnel, conceive it to be different in different media; but even if the second of these opinions be the more probable,† and thus the molecular forces have the power of altering the ether's density, yet assuredly no such power can be ascribed to gravity.

No traces of the resistance of ether are discoverable in the motions of the planets, and this has been considered as arising from their greater density and less volume; but this is subject to much

* *Astron. Nach.* tome xiii. p. 185.

† See the Author's article on the "Plane of Polarisation." *Vetanskeps-Akademicus Förhandlingar*, 1853, No. 6.

doubt. Fresnel's theory of aberration, as well as direct experiments by Fizeau,* seem to show that ether, to a certain degree, participates in a diaphanous body's motion, and that in proportion to the said body's refracting power. Thus, the ether's resistance takes place within the body itself, and must, therefore, be independent of the body's volume, but must increase in the ratio of its refracting power. But as this, on the other hand, is compensated by the greater mass which bodies of strong refracting power generally possess, it would seem that we are by no means at liberty to assume any great difference between planets and comets with respect to the ether's resistance.

Meanwhile our knowledge of the component parts of comets is extremely scanty. That they are not either solid or gaseous bodies, is evident from their property of not refracting light. They are, probably, of a powdery consistency. This we may conclude from their property of reflecting the sun-light, combined with the experience of certain observers,† that stars, seen through the nucleus itself, appear with even increased brightness. This last-mentioned phenomenon, may perhaps rest on an illusion, as others, for instance Bessel, have observed the contrary; but it may nevertheless be explained by the laws of the interference of light. For if a bright object be occulted by a body of small surface, that object may yet show itself with undiminished lustre behind the occulting body in the direction of the line of junction. Then if the occulting body be itself luminous, its light is added to that of the bright object, which is thus made to appear brighter.

As regards the assumption that the resistance varies as the square of the body's velocity, it is subject to the objection, that the notion of great or small can here only be applied to a velocity with reference to the medium in which the motion takes place; and thus that a velocity, which for air is considerable, cannot be so considered for ether. The speed with which these two separate media transmit undulations, is the best starting-point for a comparison. Thus it seems that the perihelion velocity of Encke's comet is barely one 5000th part of that of light, or about $\frac{1}{5}$ foot per second. This circumstance appears to me of especial weight, when one endeavours to reckon the resistance of ether, which again is still further modified by its probably arising within the body itself.

It would seem to follow, from the preceding considerations, that the density of the ether is throughout space the same, and that its resistance, if it really have any sensible influence, must vary as the first power of the velocity. Professor Hansen, who subjected the different hypotheses on the ether's resistance to a searching examination, has shown that the latter assumption makes but little difference in the numerical value of the resistance, but not so the

* *Comptes Rendus*, xxxiii. p. 349.
† Piazzi, Struve, Reslhuber.

former. For if we give the function (called by Encke U*) a value answering to the observed shortening of the periodic time, the perihelion distance decreases faster than it would if the ether's density increased in the neighbourhood of the sun, inasmuch as the eccentricity in that case diminishes more slowly. Encke has, indeed, with respect to the appearances of 1822 and 1832, when the comet was seen only after perihelion, uttered a suspicion, that the true law of the resistance is as yet undiscovered, and the deviations at the last appearance seem to confirm this: but whether the assumption of a constant density would represent the observations any better, cannot be decided beforehand. Still, to some extent, this would seem to be the case, since the observations of 1822 and 1832 tend to give the comet's orbit a greater eccentricity than what results when only observations before perihelion are used. They would, therefore, better harmonise with that hypothesis, as to the nature of ether, which makes the eccentricity decrease more slowly.

The objections to the hypothesis of the ether's resistance, which have been set forth in the preceding pages, have, however, awakened in my mind a lively wish to explain, by some other process, the acceleration which observations prove in the periodic time of Encke's comet; so much the more, as the celebrated proposer of that hypothesis himself remarks,—"dass das widerstehende Mittel nur als eine Form fur die nothwendig gewordene Correction anzusehen ist."†

The small planets, whose attractions have not hitherto been taken into account, have, in this respect, attracted my especial attention, and I here take the liberty of setting forth the results of the preliminary investigation that I have instituted on that subject. Even if it should appear that the observed inequality in the motion of Encke's comet cannot by this means be explained, yet every contribution to the knowledge of the mutual relations of these celestial bodies must possess at least some interest. Encke's comet will, assuredly, at some future time, throw light on the extent and masses of the small planets, and thus still further increase the number of truths wherewith it has already enriched astronomical investigations.

The number known of these bodies, from 4, which was the amount in the early part of the present century, has, within the last ten years, risen to 27. The rapidity with which these discoveries have followed one another, renders it probable that their real number is very great, although, on account of their smallness, the greater part of them are likely to remain unknown to us. Whether one adopt the Olbersian hypothesis, that all these asteroids are fragments of one large planet, or—be their origin what it may—that they form a sort of belt or ring

* [U is used by Encke for the portion of the disturbing force that arises from the resisting medium.— A. D. W.]

† *Astr. Nachr.*, xii. p. 321.

between *Mars* and *Jupiter*, it is evident that their secular perturbations, which are independent of their places in their orbits, must be the same, whether we consider them as all united in one, or disposed upon several points of a common orbit. The secular perturbations, then, of Encke's comet by the small planets, may give the approximate value of their aggregate mass.

To ascertain the mean position of the 27 asteroids' orbits, I have, from their elements, calculated the mean value of the following quantities :—

$$a\,e\cos.\varpi, \qquad e\sin\varpi, \qquad \sin\Omega\tan I, \qquad \cos\Omega\tan I,$$

and hence obtained,—

For the Asteroids.		For Encke's Comet.	
$\varpi = 65^\circ\ 1'$	$e = 0{\cdot}0555$	$\varpi_1 = 157^\circ\ 18'{\cdot}4$	$e_1 = 0{\cdot}8478$
$\Omega = 138\ 23$	$a = 2{\cdot}567$	$\Omega_1 = 334\ 29{\cdot}5$	$a_1 = 2{\cdot}222$
$I = 4\ 19$		$I_1 = 13\ 20{\cdot}7$	

where the elements of Encke's comet are, for comparison's sake, added. Moreover, if we call—

The inclination of the comet's orbit to that of the mean asteroid ..... ι
The distance of the perihelion from the ascending node ..... ω
The same angle for the mean asteroid's orbit ..... ω_1

we have

$$\iota = 17^\circ\ 32' \qquad \omega = 186^\circ\ 47'{\cdot}5 \qquad \omega_1 = 94^\circ\ 21'$$

We see, then, that the two orbits form, as it were, two links in a common chain, and that their perihelia are nearly at right angles to one another. This, combined with the large value of ι, shows that the two orbits cannot come into very close quarters with each other. On examining the asteroids' orbits separately, we find that, for some of them whose node-lines fall in the first, and perihelion in the second quadrant, an approach to the comet's orbit is possible. I have, for several of them, reckoned the angles ω and ω_1, and hence the comet's and asteroids' radii-vectores in the line of nodes ; but for only three has the difference been less than unity.

	ι	r	r'	Diff.
Irene	$18^\circ\ 48'$	2·361	2·811	0·450
Proserpine ...	15 52	3·192	2·662	0·527
Ceres.....	18 48	2·023	2·959	0·936

where r is the comet's, and r' the asteroid's radius-vector, in the line of the comet's ascending node. Hence we see that among the hitherto discovered asteroids, there is none that approaches sufficiently near to the comet's orbit to exercise any sensible influence on the comet's motion. It does not, however, hence

follow that there may not be such among the yet undiscovered. Were there one with a mean distance of nearly 2·222, it would produce a periodic inequality of a secular nature, which, in spite of the asteroids' smallness, might be sensible on the integration of the differential formulas.

To return to the secular perturbations which the asteroids cause to Encke's comet. This belongs to the more difficult class of astronomical problems, inasmuch as the projections of the two orbits on the ecliptic intersect each other. Bessel* has indeed shown how to find the secular perturbations which a planet causes to a comet, or planet with a comet-like orbit, when the disturbing force can be developed in a converging series in powers of $\frac{r'}{r}$, if r and r' signify the radii-vectores of the comet and planet respectively; and Hansen† has discussed the opposite case in which the comet's radius-vector is always less than the planet's ; but these methods both fail when the orbits cross one another.

The following method will not, then, be devoid of interest, although it must be owned that, when the mean distances of the two bodies are nearly equal, it does not lead to very rapidly converging series.

If we designate by m and m' the comet's and disturbing planet's respective masses, and their co-ordinates by

$$x, y, z, r, x', y', z', r',$$

and their distance from each other by ϱ, we obtain, as is well known, the components of the disturbing forces by differentiating the Lagrangian function,

$$\mathrm{R} = -m' \cdot \frac{(x \cdot x' + y \cdot y' + z \cdot z')}{r'^3} + \frac{m'}{\varrho};$$

but as the secular perturbations are independent of the planet's co-ordinates, we can reject the first term of this expression, whence we have

$$\mathrm{R} = \frac{m'}{\varrho}.$$

Since, moreover, in this expression those quantities only are to be retained which are independent of the planet's mean anomaly $n' t$, we may put

$$\mathrm{R}_1 = \frac{m'}{2\pi} \int_0^{2\pi} \frac{dn't}{\varrho} \ddagger$$

* *Astron. Nachr.*, xiv., p. 1.

† *Ermittelung der absoluten Störungen in Ellipsen*, 1843.

‡ [If we develope $\frac{m'}{\varrho}$ in a series of multiples of the cosines of the mean heliocentric elongation, and then cast away all the terms involving $n't$, there will evidently remain only the series of constants expressing this definite integral.—A. D. W.]

For the finding of this integral it is necessary to expand $\frac{1}{\varrho}$ in a converging series. For this purpose, if we call the angle between the radii-vectores U, we have

$$\varrho = \sqrt{(r^2 + r'^2 - 2 \cdot r \cdot r' \cdot \cos U)},$$

and can put

$$\frac{1}{\varrho} = \frac{1}{(r^2 + r'^2)^{\frac{1}{2}}} + \frac{1}{2} \cdot \frac{(2 \cdot r \cdot r' \cdot \cos U)}{(r'^2 + r^2)^{\frac{3}{2}}} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{(2 \cdot r \cdot r' \cdot \cos U)^2}{(r'^2 + r^2)^{\frac{5}{2}}} + \ldots \quad (a)$$

or generally,

$$\frac{1}{\varrho} = \frac{1}{(r'^2 + r^2)^{\frac{1}{2}}} + \Sigma \frac{1 \cdot 3 \cdot 5 \ldots (2 \cdot n - 1)}{2 \cdot 4 \cdot 6 \ldots (2 \cdot n)} \cdot \frac{(2 \cdot r \cdot r' \cdot \cos U)^n}{(r'^2 + r^2)^{n + \frac{1}{2}}}$$

where n takes all positive values from 1 to ∞.

The convergency of this series is determined by the expression $\frac{2 \cdot n - 1}{2 \cdot n} \cdot \frac{2 \cdot r \cdot r' \cdot \cos U}{r'^2 + r^2}$, which must always be less than unity, if r is either greater or less than r', and also even if $r = r'$, unless at the same time $\cos U = 1$. The series (a) may, therefore, be considered as always convergent, except in the case of a collision between the two celestial bodies, in which case, however, the differential equations also would cease to hold.

Since, for perturbations of the first order, if u' be the excentric anomaly

$$d \cdot n' t = \frac{r'}{a'} \cdot d u'$$

we have $$\frac{1}{2 \cdot \pi} \cdot \int_0^{2\pi} \frac{d \cdot n' t}{\varrho} = \frac{1}{2 \cdot \pi} \cdot \int_0^{2\pi} \frac{r'}{a' \cdot \varrho} \cdot d u' = \frac{R_1}{m'}.$$

It is not necessary for the determination of R_1 to avoid developments proceeding according to the ascending powers of the planet's excentricity e'. If, then, we put

$$\frac{1}{m'}, R_1 = \frac{V^0}{(a'^2 + r^2)^{\frac{1}{2}}} + \frac{V^i}{(a'^2 + r^2)^{\frac{3}{2}}} + \frac{V^{ii}}{(a'^2 + r^2)^{\frac{5}{2}}} + \frac{V^{iii}}{(a'^2 + r^2)^{\frac{7}{2}}} + \text{\&c.} \quad \ldots \quad (b)$$

and reject terms multiplied by e'^3 and the higher powers, we have

$$V^0 = 1$$

$$V^i = -\frac{2}{3} \cdot a' \cdot e' \cdot p - \frac{3}{4} \cdot a'^2 \cdot e'^2$$

$$V^{ii} = \frac{3}{4} \cdot a'^2 \cdot r^2 \cdot \{(p^2 + q^2) + e'^2 \cdot (4 \cdot p^2 - q^2)\} + \frac{3}{2} \cdot a'^3 \cdot r \cdot e' \cdot p + \frac{3}{4} \cdot a'^4 \cdot e'^2$$

$$V^{iii} = \frac{-75}{16} \cdot a'^3 \cdot r^2 \cdot e' \cdot p \cdot (p^2 + q^2) - \frac{15}{32} \cdot a'^4 \cdot r^2 \cdot e'^2 \cdot (25 \cdot p^2 + 2 \cdot q^2)$$

$$V^{iiii} = \frac{105}{64} \cdot a'^4 \cdot r^4 \cdot (p^2 + q^2)^2 + \frac{1}{4} \cdot a'^4 \cdot r^4 \cdot e'^2 \cdot (18 \cdot p^4 + 15 \cdot p^2 \cdot q^2 - 3 \cdot q^4)$$
$$+ \frac{105}{64} \cdot a'^5 \cdot r' \cdot e' \cdot p^3 \cdot (p^2 + q^2)$$

and so on, or if we preserve only the first power of e',

$$\left.\begin{aligned} R_1 = {} & \frac{1}{(a'^2+r^2)^{\frac{1}{2}}} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{a'^2\cdot(p^2+q^2)\cdot 2^2\cdot r^2}{(a'^2+r^2)^{\frac{5}{2}}} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{a'^4\cdot(p^2+q^2)^2\cdot 2^4\cdot r^4}{(a'^2+r^2)^{\frac{9}{2}}} \\ & + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{9}{10}\cdot\frac{11}{12}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{a'^6\cdot(p^2+q^2)^3\cdot 2^6\cdot r^6}{(a'^2+r^2)^{\frac{13}{2}}} + \&c. \quad . \quad . \quad . \\ & - \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{a'\cdot e'\cdot p\cdot 2\cdot r}{(a'^2+r^2)^{\frac{3}{2}}} - \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{3}{2}\cdot\frac{5}{4}\cdot\frac{a'^3\cdot e'\cdot p\cdot(p^2+q^2)\cdot 2^3\cdot r^3}{(a'^2+r^2)^{\frac{7}{2}}} \\ & - \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{9}{10}\cdot\frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\cdot\frac{a'^5\cdot e'\cdot p\cdot(p^2+q^2)^2\cdot 2^5\cdot r^5}{(a'^2+r^2)^{\frac{11}{2}}} - \&c. \quad . \quad . \quad . \\ & + \frac{3}{2}\cdot\frac{1}{2}\cdot\frac{a'^3\cdot e'\cdot p\cdot 2\cdot r}{(a'^2+r^2)^{\frac{5}{2}}} + \frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{a'^5\cdot e'\cdot p\cdot(p^2+q^2)\cdot 2^3\cdot r^3}{(a'^2+r^2)^{\frac{9}{2}}} \\ & + \frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\cdot\frac{9}{8}\cdot\frac{11}{10}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{a'^5\cdot e'\cdot p\cdot(p^2+q^2)^2\cdot 2^5\cdot r^5}{(a'^2+r^2)^{\frac{13}{2}}} + \&c \quad . \quad . \quad . \end{aligned}\right\} (c)$$

where the law of the development is evident.

In the foregoing expression p and q have the following meaning :—

$$r\,p = \alpha\,x - \beta\,y \qquad x = a\,.\,(\cos u - e)$$

$$r\,q = -\gamma\,x + \delta\,y \qquad y = a\,.\,\sqrt{1-e^2}\,.\,\sin u.$$

$$\alpha = \cos\omega'\,.\,\cos\omega + \sin\omega\,.\,\sin\omega'\,.\,\cos\iota,$$

$$\beta = \cos\omega\,.\,\cos\omega' - \cos\omega\,.\,\cos\omega'\,.\,\cos\iota,$$

$$\gamma = \sin\omega'\,.\,\cos\omega - \cos\omega'\,.\,\sin\omega\,.\,\cos\iota,$$

$$\delta = \sin\omega\,.\,\sin\omega' + \cos\omega\,.\,\cos\omega'\,.\,\cos\iota,$$

where, as before, ι denotes the inclination of the comet's to the planet's orbit, and ω and ω' the distance of the respective orbits' the perihelia from comet's ascending node in the planet's orbit.

In order to obtain the secular variations of the elements we must differentiate R_1 with respect to ω, ω', e, and ι, and then integrate the result so obtained with respect to the time t. Thus, for example,

$$\delta e = -\frac{a\,.\,n}{e}\,.\,\sqrt{1-e^2}\,.\int(A_0 + A_1\,.\cos u + A_2\,.\cos 2u + \&c. \ldots)\,.\,dt$$

where

$$A_0 = \frac{1}{2\,.\,\pi}\,.\int_0^{2\pi}\frac{dR_1}{d\omega}\,.\,du, \qquad A_2 = \frac{1}{2\,.\,\pi}\,.\int_0^{2\pi}\frac{dR_1}{d\omega}\,.\cos u\,.\,du, \text{ \&c.}$$

These last integrals are obtained by the application of elliptic functions. For putting

$$r = a\,.\,(1 - e\,.\cos u), \text{ and } (\tan\tfrac{1}{2}\,.\,u)^2 = \frac{\alpha}{\beta}\,.\,(\tan\tfrac{1}{2}\,.\,\psi)^2,$$

where $a^2 = \frac{a'^2 + a^2 \cdot (1-e)^2}{a'^2 + a^2}$, and $\beta^2 = \frac{a'^2 + a^2 \cdot (1+e)^2}{a'^2 + a^2}$,

we get

$$\int_0^{2\pi *} \frac{du}{(a'^2+r^2)^{\frac{2n+1}{2}}} = \frac{k}{\sqrt{\alpha \cdot \beta} \cdot (a'^2+a^2)^{\frac{2n+1}{2}}} \cdot \int_0^{2\pi} \frac{(1+\mu \cdot \cos\psi)}{\{1-c^2 \cdot (\sin\psi)^2\}^{\frac{2n+1}{2}}} \cdot d\psi \ldots (d)$$

where $k = \frac{a+\beta}{2a\beta}$, $\mu = \frac{\beta - a}{\beta + a}$, $c^2 = \frac{(a+\beta)^2 - 4}{4a\beta}$.

We have, moreover,—

$$\cos u = \frac{\mu + \cos\psi}{1 + \mu\cos\psi} \quad \text{and} \quad \sin u = \frac{\sqrt{1-\mu^2} \cdot \sin\psi}{1+\mu\cos\psi}$$

and if $1 + e \cdot \mu = f$ and $e - \mu = g$,

$$1 - e\cos u = \frac{f + g \cdot \cos\psi}{1 + \mu \cdot \cos\psi}; \qquad \cos u - e = \frac{f \cdot \cos\psi - g}{1 + \mu \cdot \cos\psi}$$

The integral, in the right-hand member of equation (d), may, as is known, be expressed by elliptic functions. Furthermore, since the value of $\frac{d R_1}{d\omega}$, and the other differentials of R_1, are composed of terms, all having the form,—

$$\frac{B}{(a'^2 + r^2)^{\frac{2n+1}{2}}},$$

in which

$$B = f(\cos u, \sin u, a, e)$$

it is easily seen that they may all be expressed by elliptic functions.

To the numerical application of the foregoing, I shall have the honour of returning on a future occasion; and for the present, therefore, confine myself to the following remarks.

In the value of the mean longitude, a term, varying as the square of the time, can arise only from the perturbations of the second order, which belong to the epoch. For if in the perturbations of the epoch of the first order, which vary proportionally to the time, we insert the variations of the elements, which are also proportional to the time, there must necessarily arise a term, varying as the square of the time.

I find, by an approximate calculation, that the variation of the eccentricity of Encke's comet, as far as it is caused by the influence of the asteroids, must, for the present, be negative; so that

$$\delta e = -t \times \text{constant}.$$

There arises, mainly through the action of *Jupiter* and *Saturn*, in the value of the epoch, a term proportional to the time, to be

* Mr. Wackerbarth, in a note which we regret we have not room to insert here, has given the analysis of the process by which this transformation is effected.—Editor.

added to the comet's mean anomaly $n\,t$. If in this the value of $\delta\, e$ be inserted, there must result a term diminishing with the square of the time. Whether, however, that term be of sufficient amount to account for the observed circumstances in the comet's period, is another question, which mainly depends upon the value we assign to the asteroids' masses and the eccentricity of the assumed common orbit. Against this, it may be urged, that observation does not indicate any very considerable change in the eccentricity of the comet's orbit; and, moreover, that the value of the asteroids' masses cannot unconditionally be assumed greater than what answers to their influence on the nearest planets.* But, on the other hand, it is not necessary to ascribe to that cause alone the whole of the observed diminution of the period, since that can partially be explained by the perturbations of the first order, which the asteroids cause to the period or epoch; and besides, the elements of the orbit themselves must be somewhat altered if the ether's resistance be ignored. The necessity of a correction for the period of Encke's comet may, indeed, be considered as fully established by observation, but not so the form of that correction. For as far as that proof rests upon the apparitions of 1805, 1795, and 1786, the observations are better satisfied by assuming a periodic correction, embracing about seventy-two revolutions of the comet; though, it must be owned, that this does not in any way facilitate the explanation of the phenomenon.

In the foregoing investigation, I have assumed that the resisting medium is the same ether, whose existence is presupposed as necessary for explaining the phenomena of light and heat: one might also suppose that the resisting medium is the last stratum of the sun's atmosphere, which, by its reflexion, probably causes the so-called "corona" in a total eclipse. It happens, however, in this case, that that atmosphere must participate in the sun's rotation, and thus its resistance must be insensible.

* Since the above was written, the Author has had the opportunity of examining, in the *Comptes Rendus*, Nov. 28, 1853, an interesting article, by M. Le Verrier, on the masses of the small planets. Le Verrier considers that, with regard to the relative positions of the common asteroidal orbit, and *Mars'* orbit, the second term of the series, expressing the motion of *Mars'* perihelion, rises to $\frac{1}{4}$ of the first term of the same series. If, then, under this supposition, we take the aggregate mass of all the small planets as equal to the earth's mass, they must, according to M. Le Verrier, cause an inequality in the heliocentric longitude of *Mars'* perihelion, amounting to 11″ in a century. Now, as an inequality of one fourth of that amount could not assuredly have escaped the notice of observers, M. Le Verrier concludes that the sum of the masses of the small planets cannot surpass $\frac{1}{4}$ of the earth's mass. Even if with time this fraction should undergo some considerable modification, still a possibility is thus afforded for determining the limit of the masses of the small planets, which cannot but be of the highest utility in the question of the influence of these masses on Encke's comet.

The following communication relative to the elements of Comet II. 1854, has been received from Mr. Powell. It will be seen that it includes the application of the corrections for aberration and parallax, which were not taken into account in his previous investigations.

Correction of the Elements of Comet II. 1854.
By E. B. Powell, Esq.

Greenwich M.T. April	Long. Observed, Corr. for Aberr. and Parallax.	Corr. Applied.	Lat. Observed, Corr. for Aberr. and Parallax. (South)	Corr. Applied.	
	° ′ ″		° ′ ″		
8·06067	49 51 43	+47	5 22 57	+33	The sun's real Long. was used, instead of that in the *Nautical Almanac.*
13·087693	59 0 44	+41	12 33 45	+26	
27·08739	75 31 8	+31	24 29 4	+16	

"The aberration was obtained from the diurnal motion and distance of the comet by means of a table in Vince's Astronomy; and the parallax was found roughly by taking the height and longitude of the nonagesimal from a large celestial globe, and then applying calculation.

"On pushing further the approximations, I found

D, the perihelion distance = ·2773579
E, the time of perihelion passage = March 24·010625

which afforded the errors of anomaly,

$u - \nu = +1''$, from 8th to 13th, in an angle of 10° 41′
$u' - \nu' = -16''$, from 8th to 27th, in an angle of 27 11

"If D be diminished by ·00003, the errors become

$$u - \nu = +6'' \quad \text{and} \quad u' - \nu' = -6''$$

and therefore less on the whole; but I have no doubt of the observation of the 27th being more inaccurate than those of the 8th and 13th; and, consequently, I am inclined to prefer the value of D first mentioned.

"The resulting elements are

D = ·2773579
E = March 24th, ·010625, or 24th, $0^h\ 15^m{\cdot}3$
Inclination of Orbit = 82° 26′ 43″
Long. ☊ = 315 30 11
Long. γ on ecliptic = 167 56 19
Motion retrograde.

"Working out the comet's place on the 18th,

The error of long. = about 10″
The error of lat. = about 4

The very great obliquity of the orbit affects the accuracy of the determination of the inclination, the angle in the numerator of the expression,

$$\cos i = \frac{\sin(\varphi - \varphi')}{\sin(v' - v) \cdot \sec\lambda \cdot \sec\lambda'}$$

being only 4° 19′ 13″, using the interval from the 8th to the 27th.

"It may be remarked that Pontécoulant's modification of La Place's method of approximating to the two fundamental elements gives an error of about —·0056 in the perihelion distance, and one of about —·0078 days, or —11 minutes, in the time of perihelion passage.

"*June 5th*, 1854."

Observations on an important Phenomenon observed with regard to the Hill of Santa Lucia, situated in the City of Santiago de Chile. By Don Carlos Moesta.

"The summit of the hill of Santa Lucia, situated in the capital of the Republic of Chili, is 630 mètres above the level of the sea, and about 60 above the city. Its greatest base is from N. to S., measuring some 5 cuadras (the cuadra is 150 varas square); its perpendicular line scarcely 2 cuadras.

"The rock of the hill, at first sight, appears to be a basalt; but, on examination, Señor Domeyko has determined it to be a variety of metamorphic porphyry, which he calls 'porfido abijarrado.' The columns of the rock have in various parts not only different directions, but various inclinations. However, the columns in the most southern part of the hill run W. 60° N. with an inclination of 36° to S.E.; the columns at the N. are nearly horizontal, with direction to W. a little to N. The upper part of the hill N.E. and S. is covered with a layer of earth, and not much vegetation; part of it also has broken stones (angular); but to the W., where there is a quarry, the heads of the columns are seen, and it is here where the hill is very steep. It has been necessary to enter into the above details to better understand the phenomenon I am about to refer to.

"In the N. part of the hill was erected, in 1849, the Astronomical Observatory, and it was then believed this was the most favourable position. I here only allude to the house which contains the transit instrument. The axis of the instrument rests on two blocks of stone of $6\frac{1}{2}$ feet, which are placed in solid masonry upon the firm rock, without touching any broken or soft parts of the rock.

"The rock that serves as foundation to this masonry, as I have already stated, is of the before-mentioned columns of porphyry; the direction of these is about in a line with the axis of the instrument, and with but little inclination. In the construction of the instrument, as well as in the fixing it, the greatest care

was taken, so as to compensate for all exterior influences of variation of temperature, pressure of the air, &c.

"Independently of all this, Mr. Gilliss* has observed from the period of the erection of the observatory, that the end of the axis to the E. was constantly elevating itself, and that it was found necessary to lower this end from time to time, so that the instrument might be maintained in the most convenient position for observing. This phenomenon has been observed since then until the present time, and clearly shows that the block on which rests the extremity of the axis to the east is being raised above the surface of the earth, and in such a manner, that if we add the small quantities, the other end has been lowered since the erection of the observatory in 1849 (these observations of Moesta's were made in March 1853), the difference will be *a quarter of an inch.*

"So interesting a phenomenon induced me to examine if this elevation was effected suddenly, that is to say, by movements caused by earthquakes, which are scarcely sensible on the surface of the earth, or by a motion slow but gradual.

"I proposed to level the instrument every 12 hours, with all possible exactitude; and in carrying on these operations, I observed another phenomenon, not less interesting than the first. I observed that in taking these levels, with the large level of the transit, the position of the axis was subject to oscillations dependent on temperature.

"Here are the observations I refer to:—

Days. 1853.	Height of Axis to the East.	Height of Axis to West.	Error of Obs.	Prob. Error of Result.	Temperature, Fahr.	State of Atmosphere.	
Mar. 4	0·007	...	0·190	0·064	70·7	Clear and calm.	
5	...	1·049	0·246	0·058	91·6	—	—
	0·063	...	0·037	0·009	70·0	—	—
8	...	1·794	0·074	0·022	84·0	—	South wind.
	...	1·052	0·206	0·069	62·5	—	Calm.
9	...	2·227	0·067	0·023	89·0	—	—
	...	1·210	0·121	0·040	69·7	—	—
11	...	2·431	0·220	0·065	88·5	Cloudy	—
	...	1·803	0·240	0·064	68·0	Clear	—
12	...	2·369	0·204	0·061	84·5	Cloudy;	strong S. wind.
14	...	2·695	0·050	0·016	68·0	—	Calm.
	...	2·475	0·029	0·009	62·5	—	—
15	...	3·041	0·151	0·045	77·5	Clear	—

"This table contains, independently of the difference of elevation of the two extremities of the axis, the temperature and state of the atmosphere; and so that we may have an idea of the exactitude of the observations, I have calculated the medium

* Chief of the United States Astronomical Expedition.

errors of the observations, as well as the probable. Observe that the observations are made at intervals of 12 hours. It is sufficient to look at the table to be convinced that the end of the axis to the E. rises and falls with elevation or depression of temperature; and as, by the construction of the instrument, the temperature influences both ends of the axis in a similar manner, we have to look for the cause of that oscillation away from the instrument; *this cause can be no other than the expansion and contraction of the columns of porphyry*, in which is placed the block that supports the west extremity of the axis; expansion and contraction caused by the heat of the sun during the day and the cold at night.

"Thus, for example, on the 4th and 5th March, the axis rose to the west,

1″·056

whilst the temperature rose

70°·7 to 91°·6 F.

"The same axis fell afterwards

1″·112

"Temperature depressed

91°·6 to 70° F.

"Little more or less we find this same result in the second group of observations; and if the difference of level in these two groups have not the same relation, with the corresponding differences of temperature, less have we of coincidence in the following group. The 11th and 12th of March the difference of level was only

0″·628

Temperature depressed from	88°·5 to 61° F.
And it is	0″·596
Temperature rising	68° to 84°·5 F.

"But this inequality of the differences of level for equal differences of temperature is a proof most truly, that the expansion of the rock produced by the sun's heat is the cause of the rising of the hill.

"The first two groups of observations show that the difference of level was a little more or less than

1″

the difference of temperature rising, medium height, to 20°; whilst in the third group the difference of level is only

0″·628,

notwithstanding the difference of temperature was the same.

"But the last column of the table indicates that at the same time, during the first observations, the sky was clear, and cloudy

during the others. It is evident that, with a clouded sky, the hill or rock cannot become so heated as when it receives the direct rays of the sun; and, consequently, the difference of the temperature of the air and that of the rock must be less in the first case than in the second.

"The temperature of the rock will also depend upon the wind; for example, it will be less when blowing strong from S., or when it is calm. Supposing in both cases the temperature of the air to be the same. Thus, then, there cannot be a constant relation between the differences of level and the difference of temperatures; but it is more than probable that we shall find determined the differences of the temperature of the same rock, in place of the temperature of the air.

"The last group of the observations presents an example in which the level, during 12 hours, remains constant: this fact coincides with the very small change of temperature; in effect, the temperature for the difference being (on the 14th) only 5°·5; the difference of level was not above

0″·22

"According to these observvations, there cannot be a doubt that the heat of the sun is the cause of the phenomenon in question; and now it only remains to inquire, how comes it that there is only a partial rising of that part of the hill situated to the west, which elevation is indicated by the level.

"To resolve this question, we must again revert to the topography and geology of the Hill of Santa Lucia.

"I have said that in the W. part of the hill the porphyritic columns are exposed to the immediate influence of the sun, whilst in other parts the hill is covered with a layer of vegetation and broken rock, which protects the said columns from the sun's heat. Moreover, these columns have a N.W. direction, with a strong inclination to the west; so that being exposed from 12 P.M. to the evening to the sun's rays, which fall almost perpendicularly upon the heads of the columns, will necessarily suffer a greater expansion than the great body of the hill situated to the N. and E., which is shaded.

"And thus we must be struck with this wonderful power or force of the sun which causes to rise and fall *periodically* this enormous mass of firm and hard rock analogous to that other force which causes to rise and fall *periodically* the column of mercury in the barometer.

"It will now easily be conceived that in the vicinity of Santiago there could not be a worse place for an observatory than the Hill of Santa Lucia, it being of the greatest importance to have such established at a point where the atmospheric influences have the least possible effect on the instruments."

On the Phenomenon seen during the Total Eclipse of the Sun of November 30, 1853.

(*Extract of a Letter from Capt. Shea.*)

"On perusing the account of the total eclipse of the sun on the 30th November, 1853, at Ocucaja, in Peru (communicated by Admiral W. H. Smyth), in No. 8, June 9th, 1854, of the Royal Astronomical Society's *Monthly Notice*, I find, on referring to my register of the spots passing over the sun's disc, that the spot seen here on the 24th of November, 1853, would occupy the same position in the solar hemisphere as the 'rose-coloured protuberance' described in the account above referred to."

Captain Shea further remarks, that the observations of the total eclipse of the sun, July 28, 1851, appear to him to strengthen the opinion of the identity of the solar spots with the rose-coloured protuberances.

Tables for Facilitating the Determination of the Latitude and Time at Sea by Observations of the Stars. By Charles F. A. Shadwell, Esq., F.R.A.S., Captain Royal Navy. New edition, 8vo. London, 1854.

This new edition of a work well known to the naval profession is distinguished by various improvements, which have a tendency to promote its useful character. Several additional couplets of stars are inserted in Tables I. and II., which, it may be stated by way of explanation to those who have not seen the work, are computed with a view to abbreviate the process for finding the latitude by "Simultaneous Altitudes," or, in other words, by simultaneous observations of the altitudes of two stars.

The author has pointed out the facility with which, under certain favourable circumstances, the same method may be employed for finding the Time, and he has computed a new Table (Table III.) with a view to the promotion of this object. He has also computed a fourth Table adapted to the method "for finding the latitude by the combined altitudes of two stars taken at an interval of time equal to the difference of their right ascensions."

In reference to the last-mentioned method, which was originally given in Lynn's Navigation, but appears to have subsequently fallen into oblivion, the author remarks that, "since the observations required are in themselves very simple, since the practical fulfilment of the requisite condition as to time demanded by the problem is not attended with any particular difficulty, and since the actual computation may be materially facilitated by the use of previously prepared constants depending on the declinations of the stars, there does not seem to be any valid objection to the revival of this problem, and to again submitting it to the notice of the scientific navigator. Moreover, since the period during which the observations may be

made is not restricted to any precise moment, as in the case of meridional altitudes, advantage may frequently be taken of the favourable circumstances of morning and evening twilights, when the distinct definition of the sea-horizon is conducive to the accuracy of the results; a recommendation analogous to that which attends the method of Simultaneous Altitudes."

Leçons de Cosmographie, Rédigées d'après les Programmes Officiels. Par H. Faye, Membre de l'Institut, &c. &c. Deuxième edition. 8vo. Paris, 1854.

This work contains a lucid exposition of the more elementary principles of astronomy adapted to the present state of the science. The early appearance of a second edition is a sufficient proof of the esteem in which the work is held in France.

Sitzungsberichte der Kaizerlichen Akademie der Wissenschaften. Band XII. (Jänner.) Wien, 1854.

This publication contains three astronomical papers, which it may be occasionally desirable to consult. The first of these is by M. Hornstein: it contains a determination of the orbit of Comet I. 1853, founded upon the totality of the observations of the comet, extending from March 6 to April 11. The second is by M. Littrow, and refers to the points of resemblance which subsist between the orbits of the various bodies composing the solar system. The third, by M. Oeltzen, is devoted to a comparison of the Zone observations of Bessel and Argelander.

Astronomical Observations made at the Radcliffe Observatory in the year 1852. By Manuel J. Johnson, M.A., Radcliffe Observer, Vol. XIII. Oxford, 1854.

The Meridian Observations in this volume, with few exceptions, relate to the Circumpolar Catalogue. The author states that another year will be required to complete the latter on the system laid down of not admitting any star which has not been observed in at least two different years. "This system," he remarks, "though it has prolonged our task, has been found so useful a check against errors and confusions of all kinds, as to afford ample compensation for delay." The author announces that an investigation of the parallaxes of 61 *Cygni* and 1830 *Groombridge*, founded upon observations with the heliometer, will appear in the next volume, which will be shortly published.

Informe sobre las Observaciones hechas durante el Eclipse Solar de 30 Noviembre de 1853, presentado al Senor Ministro de Instruccion Publica. Por Carlos Moesta. Santiago de Chile, 1854.

This account of the total eclipse of the sun of November 30, 1853, does not differ from that communicated to the Society in the course of last summer by Admiral Smyth, and of which a translation was given in the *Monthly Notice* for June 1854 (Vol. XIV., p. 225), except that it is accompanied by a drawing of the appearance of the eclipse which some persons may be desirous of inspecting. It may be remarked, however, that the details of the drawing exhibit a satisfactory agreement with the description given in the text.

Astronomische Waarnemingen gedaan ter bepaling der geografische ligging van Batavia door S. H. De Lange en G. A. De Lange, geografische Ingenieurs von Nederlandsch Indië.

(*Communicated by the Astronomer Royal.*)

This Paper, as the title imports, contains the details of a series of observations instituted with a view to determine the longitude of Batavia. These observations consist chiefly of determinations of the zenith-distance of the moon, and of the right ascensions of the moon and moon-culminating stars. They were commenced in 1851, but were interrupted by the unfavourable state of the weather in the month of October in that year, and soon afterwards by the return of MM. De Lange to Manado. They were, however, resumed in May 1853, and continued to be prosecuted pretty regularly till towards the close of the same year.

The observatory consisted of a hut built of bamboos for the occasion, which could be opened with facility in every direction. The instruments with which the observations were made were a universal instrument by Pistor and Martin of Berlin, and a similar instrument by Repsold.

It was found inconvenient to erect a meridian mark; a bamboo was merely fixed in the direction of W.S.W. at a distance of 200 yards from the place of observation; its azimuthal deviation from the meridian having been determined with the utmost possible accuracy. In determining the error of collimation, the method of making one of the telescopes serve as a collimator to the other was very frequently employed. The adjustment of the telescopes upon each other was very conveniently effected.

In the focus of the telescope two vertical wires were placed very near to each other, which could be very accurately brought upon the wires of the telescope employed as the collimator.

In order that an opinion of the value of the observations may

be formed, a comparison is instituted between the results for right ascension obtained by the two observers on every night of the moon being observed independently by each of them. The following are the outstanding differences:—

s	s
−0·06	+0·13
−0·17	+0·03
+0·06	−0·42
−0·22	+0·56
+0·04	−0·24
−0·29	+0·05
−0·09	−0·16
+0·01	−0·05
−0·05	+0·10
+0·07	−0·55
	−0·28

The author estimates the probable error of an observation of right ascension to be 0s·92, a quantity which, however, he is inclined to think exceeds rather than falls short of the true value.

"The author then proceeds to give an account of the operations of himself and his colleague for determining the longitude from the observed zenith-distances of the moon. The method pursued by them in this instance was suggested by Kaiser, who remarks that it is only applicable in practice when the moon's latitude is small, since the variation in right ascension then takes place almost wholly in altitude.

"In the foci of both instruments," says the author, "I inserted two additional wires on each side of the double horizontal wires which had been already introduced. The instrument being adjusted with all possible precision, the telescope was directed to the moon, when the time of the passage of her limb over each of the horizontal wires was accurately noted and the level of the vertical circle read off. By the aid of a slight previous calculation, it was easy to find the instant of time (and also the corresponding azimuth) when a star in the vicinity of the moon should attain the same altitude. The instrument was then placed in the known azimuth (the vertical circle retaining a fixed position), and the star being brought into the field of view of the telescope by its diurnal motion, the time of its passage over each of the horizontal wires was observed and the level read off. The altitude of the star for the time of observation may be accurately found by calculation, and it is manifest that the apparent zenith-distance of the moon's limb will be equal to the computed zenith-distance of the star, subject to a small correction for the variation of the level corresponding to the interval between the two observations. The following are the advantages of this method:—

"1°. A small error in the determination of the time does not exercise any sensible influence.

"2°. The effect of refraction is eliminated.

"3°. The method is independent of the unavoidable errors which the observation of zenith-distances entails upon the reading off of the vertical circle. It is also independent of the errors of the instrument, as well as those of the micrometer screws, and of the error of collimation.

"4°. The observation may be repeated several times."

The following process was employed for deducing the longitude from the observations:—

With an assumed longitude ($7^h\ 7^m\ 37^s$ East) the Greenwich mean time of the observation of the moon's limb was found, and the corresponding right ascension and declination of the moon calculated from the *Nautical Almanac*.

From these data the zenith-distance of the moon's centre was finally calculated, which, if the assumed longitude was correct, ought to agree with the observed zenith-distance. If no such agreement was found to subsist, the difference of the two quantities served to indicate the difference of longitude which would result by repeating the calculation with a longitude greater or less by 10^s than the longitude assumed. The author remarks that the agreement of the results obtained in the same night by this method leaves nothing to be desired, and that the probable error of an observed zenith-distance does not amount to more than $1''{\cdot}3$, which on an average agrees with an error of $2^s{\cdot}6$ in the longitude deduced from it, a degree of accuracy, he adds, which is not to be obtained by any other method with so small instruments.

The results of the observations are then given. The longitude for each observed zenith-distance is computed, and also the small difference which would result if the assumed longitude were increased by 10^s. The transit-observations next follow with the individual values of the longitude deduced from them. The Paper concludes with the equations of condition for the correction of the longitude based upon a few observed occultations. The method of calculation employed is that given by Professor Challis in the *Nautical Almanac* for 1854.

"*Telescopic Appearance of the Planet Venus at the time of her Inferior Conjunction, February* $28^d\ 1^h\ 45^m$, 1854. By John Drew, Ph. D.

"Having ascertained from the *Nautical Almanac* that on this occasion the heliocentric latitude of *Venus* was nearly at its maximum, I determined to watch her if possible at the time of conjunction; and, as the sky was particularly clear, I succeeded in taking observations of the planet both before and after the occurrence.

"Measurement with the position wire micrometer gave, as nearly as it would admit, $3''$ as the breadth of the illumination: more than half of her circumference was visible, but its extent I

regret that I did not ascertain. On a similar occasion (May 1849) the distance from one cusp to the other was found by M. Mädler, making use of the Dorpat refractor, to be no less than 240° (*Astronomische Nachrichten*, No. 679), whence he deduces 43′ 7″ as the amount of horizontal refraction in *Venus*, or greater than that of our atmosphere by about one-sixth.

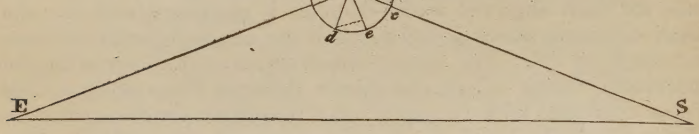
"With the object of ascertaining how much of the breadth of the crescent was due to refraction, I calculated the breadth of that portion of her surface which only would be visible, supposing the planet was not surrounded by an atmosphere, and found it to be 0″·67. The difference between this and 3″ is, I apprehend, caused by the refraction of the planet's atmosphere.

"I subjoin the calculation and a drawing of the appearance of the planet at the instant of her inferior conjunction.

	°	′	″
Heliocentric latitude of Venus	3	22	32
Geocentric latitude	8	48	10
Illuminated portion = the sum	12	10	42

"The versed sine of this last quantity is ·0225: hence ·01125 of her diameter (59″·6) = 0″·67 will be the breadth at the widest part of the illuminated crescent.

"The observations were taken at my observatory, Southampton, with a 5-foot refractor by Dollond.



"Let V be *Venus;* E the Earth; S the Sun; *a d* the boundary of light and darkness; *d e* the illuminated portion, as seen from E = *b c*; but *b c* = ∠ E + ∠ S = geocentric lat. + heliocentric lat."

Contribution to the Theory of Elliptic Functions. By Carl John Malmsten, Professor of Mathematics in the University of Uppsala.*

In this paper the author gives a new demonstration of the three well-known relations connecting the elliptic function of the third order

$$\Pi(n, k, \varphi) = \int_0^{\varphi} \frac{d\varphi}{(1 + n \sin^2 \varphi)\, \Delta(k, \varphi)}$$

* Communicated by A. D. Wackerbarth, Esq.

with those of the first and second order

$$F(k, \varphi) = \int_0^{\varphi} \frac{d\varphi}{\Delta(k, \varphi)}$$

$$E(k, \varphi) = \int_0^{\varphi} \Delta(k, \varphi)\, d\varphi$$

The author remarks that Lagrange established the relations between these functions by a process mainly founded on the differentiation of the parameter n; but he shows that the same object may be effected, independently of the differentiation of n, by means of the differential equation of the second order which is satisfied by $F(k, \varphi)$ and $E(k, \varphi)$.

Elements of Euphrosyne. By M. Winnecke.

Epoch, Sept. 1·0, M. T. Berlin.

			Mean Equinox,
M	=	299° 0′ 3″·4	
ϖ	=	95 13 45·1	1855·0
Ω	=	31 11 59·9	
i	=	26 53 26·0	
φ	=	13 15 46·2	
μ	=	622·091	
Log a	=	0·504102	

These elements are calculated from the Washington observation of September 2, and the Berlin observations of October 1 and 31.

Elements of Pomona. By M. Bruhns.

Epoch, 1854, Nov. 0·0, M. T. Berlin.

			Mean Equinox,
M	=	206° 32′ 27″·6	
ϖ	=	195 46 56·0	1855·0
Ω	=	220 44 20·5	
i	=	5 39 2·9	
φ	=	5 29 27·6	
μ	=	853·694	
Log a	=	0·412470	

These elements are calculated from the Paris observation of October 28, and the Berlin observations of November 6 and 14.

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